UNIVERSITY COLLEGE TATI

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET	
COURSE CODE	: FGE 1123
COURSE	: MATHEMATICS II
SEMESTER/SESSION	: 2 – 2022/2023(JULY INTAKE)
DURATION	: 3 HOURS

Instructions:

1. This booklet contains **5** questions in SECTION A, **3** questions in SECTION B and **2** questions in SECTION C. Answer **ALL** questions.
2. All answers should be written in answer booklet.
3. Write legibly and draw sketches wherever required.
4. If in doubt, raise your hands and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 6 PRINTED PAGES INCLUDING COVER PAGE

SECTION A (50 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

Evaluate:

a) $\lim_{x\to -2}(3x^2 - x + 1)$ (2 marks)

b) $\lim_{x\to 3}\left(\frac{x^2 - x - 6}{x - 3}\right)$ (3 marks)

c) $\lim_{x\to \infty}\left(\frac{1+x}{1-x}\right)$ (3 marks)

d) $\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}$ (4 marks)

QUESTION 2

Differentiate each of the following functions.

a) $y = 5x^4 - 3x^3 + \frac{3}{7x^2}$ (3 marks)

b) $y = 10\ln|x| + e^{3x-1} - \sqrt{x}$ (3 marks)

c) $y = (2x-1)^5 - \cos 3x + \sin(11-x)$ (3 marks)

d) $y = \tan(2x^3 + 1) + 3e^{2x} + 7$ (3 marks)

QUESTION 3

Integrate each of the following functions.

a) $\int\left(10x^5+5x^2+x\right)dx$ (2 marks)

b) $\int\left(\frac{1}{x-3}+\frac{2}{4x+1}\right)dx$ (2 marks)

c) $\int\left(\frac{7}{2}e^{4x+1}-\frac{3}{e^x}\right)dx$ (2 marks)

d) $\int\left(\sec x-8\tan 4x\right)dx$ (2 marks)

e) $\int\left(3x^{\frac{2}{3}}-\sqrt{x}\right)dx$ (2 marks)

QUESTION 4

Differentiate the following functions using the given techniques.

a) $y=\left(x^3-x\right)^4\sin 5x$ (Use product rule) (4 marks)

b) $y=\frac{\ln\left(x^2+1\right)}{e^{3x}}$ (Use quotient rule) (4 marks)

QUESTION 5

Integrate the following functions using the given techniques.

a) $\int x\cos 5x^2\,dx$ (Use substitution method) (4 marks)

b) $\int\frac{4x}{e^{4x}}dx$ (Use by parts method) (4 marks)

SECTION B (30 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

Find $\frac{dy}{dx}$ for the following functions.

a) $x=\ln(t+4)$, $y=\frac{2}{t+1}$ (4 marks)

b) $5x^2+2y-xy^2+x=16$ (8 marks)

QUESTION 2

Solve $\int \frac{10+x}{x(x^2-1)}dx$ using the appropriate method. (8 marks)

QUESTION 3

Find the equation of tangent and normal line to the graph, $f(x)=\frac{5x}{(x-6)^2}$ at the point $x=-1$.

(10 marks)

SECTION C (20 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

Given that $f(x)=2x^3-9x^2+12x-3$.

a) Find the stationary points on the curve. (4 marks)

b) Determine the maximum and minimum points. (3 marks)

c) Find the inflection point. (2 marks)

d) Sketch the graph. (1 mark)

QUESTION 2

a) Sketch the graph of a quadratic function, $y=5-(x-2)^2$ and a line, $y=x-3$ on the same axis. Then shade the region between both graph. (5 marks)

b) Calculate the area enclosed by the curve and the line in (a). (5 marks)

----------End of question----------

FORMULA

$$\frac{d}{dx}(k)=0$$

$$\frac{d}{dx}(x^n)=nx^{n-1}$$

$$\frac{d}{dx}\left[f(x)\right]^n=n\left[f(x)\right]^{n-1}f'(x)$$

$$\frac{d}{dx}(\sin u)=u'\cos u$$

$$\frac{d}{dx}(\cos u)=-u'\sin u$$

$$\frac{d}{dx}(\tan u)=u'\sec^2 u$$

$$\frac{d}{dx}(e^u)=u'e^u$$

$$\frac{d}{dx}(\ln u)=\frac{u'}{u}$$

$$\frac{d}{dx}(uv)=uv'+vu'$$

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{vu'-uv'}{v^2}$$

$$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$$

$$y=mx+c$$

$$y-y_1=m(x-x_1)$$

$$\int k\,dx=kx+C$$

$$\int x^n dx=\frac{x^{n+1}}{n+1}+C,\ \text{for } n\neq -1$$

$$\int (ax+b)^n dx=\frac{(ax+b)^{n+1}}{(n+1)(a)}+C,\ \text{for } n\neq -1$$

$$\int \cos(ax+b)\,dx=\frac{1}{a}\sin(ax+b)+C$$

$$\int \sin(ax+b)dx=-\frac{1}{a}\cos(ax+b)+C$$

$$\int \sec^2(ax+b)dx=\frac{1}{a}\tan(ax+b)+C$$

$$\int e^{ax+b}dx=\frac{1}{a}e^{ax+b}+C$$

$$\int \frac{1}{x}\,dx=\ln|x|+C$$

$$\int \frac{1}{ax+b}dx=\frac{1}{a}\ln|ax+b|+C$$

$$\int udv=uv-\int vdu$$